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EUROPEAN QUANTUM FUTURE ACADEMY 2020

SECOND PROBLEM SET

4) Consider a pair of photons of different colors. Red one located in a secret laboratory on Saturn and blue one being in possession of an experimentalist on Pluto. Let us denote the joint quantum state of their polarisation degrees of freedom as ρ.
    a) Assume experimentalists are communicating together and jointly apply the same unitary transformations of the polarisation degrees of freedom on their respective photons, i.e. transformations of type $U \otimes U$ where U is unitary performed in a given lab. Find a pure state ρ_0 that does not change under any synchronized unitary transformation.
    b) Consider the photons are in the state ρ_0. Alice on Saturn registers that photon passed her horizontally oriented polarizer and performs the measurement of circular polarisation. Bob on Pluto performs also the measurement of circular polarization of his photon. Suppose Alice has registered that the photon is left-polarized. What is the probability that Bob will find the same outcome of his measurement? How is this probability going to change if Bob uses vertically oriented polarizer before his measurement of the circular polarisation?
    c) Consider that experimentalists (Alice and Bob) agreed not to implement particular unitary transformation, but they agreed to transform the whole ensemble of pairs by randomly applying only Pauli operators, but in a synchronized way. In other words, one of them uniformly selects one of four possibilities: 0 if no transformation is applied, 1 if $U=\sigma_x$, 2 if $U=\sigma_y$, and 3 if $U=\sigma_z$. Assume the initial state is ρ. The selected random choice is communicated to the other side, thus, the same transformation is applied on both photons. Evaluate the final state ρ' of the polarisation degrees of freedom of the ensemble of photon pairs. Compare the overlaps $\mathrm{tr}[\rho\rho_0]$ and $\mathrm{tr}[\rho'\rho_0]$.

Please send your solutions to mario.ziman@savba.sk before 10/07/2020, but the sooner, the better. Specify your name and faculty. It is recommended to send answers of each problem independently.